THE STRATEGIC ROLE OF ASSET SPECIFICITY IN SHAPING INDUSTRY STRUCTURE

Francisco Ruiz-Aliseda[♣]
INSEAD

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Abstract

Empirical examination of certain industries shows heterogeneity on the degree of specificity of the assets employed by different firms. This paper studies why such asymmetries may be observed based on the strategic implications of asset specificity in an uncertain environment. To this end, we present a new class of real options games with a binary entry mode in which exit is allowed. Thus, firms can choose the timing of (dis)investment in an asset whose degree of specificity is selected when entering the market, which affects incentives to wait and see, and hence preemptive incentives. We analyze why ex ante identical firms not only choose to enter and exit at different dates, but usually choose to invest in assets with different redeployment/resale values.

Key words: Commitment, Flexibility, Bayesian Real Options, War of Attrition, Preemption, Rent Equalization Principle, and Markov Perfect Equilibrium.

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[♣] Economics and Political Science, INSEAD. E-mail: fran.ruiz@insead.edu. I am very grateful for valuable insights to Sandeep Baliga, David Besanko, Timothy Van Zandt, Jianjun Wu and Peter Zemsky, as well as seminar participants at Universidad Carlos III de Madrid and WZB in Berlin. The author gratefully acknowledges financial support from Fundación Rafael del Pino. The usual disclaimer applies.

1. INTRODUCTION

In today's business world, firms have to deal with a significant amount of uncertainty when making their long-run –and even short-run– strategic decisions. Rapidly changing consumer preferences, increased global competition, or the quick pace of technological change in both input and output markets are apparently inexhaustible sources of new threats and opportunities to firms competing for and in the marketplace.

Given such widespread uncertainty, firms have an incentive to follow investment strategies that preserve their flexibility to react to changing market conditions. To a large extent, this explains the usage of assets such as general-purpose technologies or multipurpose plants in industries as diverse as the chemical, food processing or automobile industries.[1] Firms in industries with few actors weigh more aspects when undertaking large investments, though. The choice of assets has strategic implications for the dynamics of industry structure, as studied by a large branch of the IO literature, starting with Dixit (1980). For example, a firm that preempts its competitors and pre-commits not to carry out certain actions can force them to respond softly, thus improving its payoff. In the context of the dynamic selection of assets, strategic commitment to stay in a certain market can be achieved by investing in "sticky" factors (e.g., specific human capital or know-how) whose redeployment value is residual. Investment costs that are largely sunk make preemptive moves credible and thus induce rivals to delay their entry.

In many occasions, firms have the opportunity to invest in an array of assets characterized by different redeployment or resale values. For instance, a firm in the carbon fiber industry can build either a plant specialized in producing high-grade or low-grade carbon fiber or a multipurpose plant that can switch production from one grade to another at a relatively low cost. Indeed, empirical examination of certain oligopolistic industries, such as the fine-paper industry by Upton (1997), shows heterogeneity on the degree of flexibility of the assets employed by different firms, despite the fundamental process required to manufacture products is essentially homogenous across firms. One of the objectives of this paper is to provide reasons why such asymmetries may be observed based on the strategic implications of asset specificity in an uncertain environment.

[1] For example, Upton (1994) attributes the success of U.S. Robotics before the 90's to an asset bundle that allowed it to exit dying (or temporarily lame) businesses and move into new ones at a low cost.

So far, much is known about the effect of resource specificity on the choice of a firm's organizational form, and relatively little is known about the strategic implications for industry structure. Although the strategic value of asset specificity as a competitive weapon *in* oligopolistic product markets seems well understood,[2] its strategic implications for competition *for* oligopolistic markets have been substantially ignored. In a broad sense, the main purpose of this paper is to provide a theoretical examination of competitive dynamics –i.e., when and how industry structure is formed– when different asset choices are available to firms. Such analysis also allows us to identify the precise reasons why firms may exhibit a preference/dislike for resource specificity in a game-theoretic setting with endogenous order of entry and exit as well. In this sense, we can also draw predictions about the nature of assets in which first- and second-entrants invest, thus opening new avenues for empirical research within this largely unexplored field.

In this paper, we model timing competition between two ex ante identical firms that can choose the degree of specificity of their resources as well as the date at which to invest in such assets given a market whose evolution is uncertain. In particular, the market grows until a random maturity date and declines thereafter, which creates an incentive to update information by waiting to invest as in the real options tradition (see e.g. Dixit and Pindyck 1994 or Trigeorgis 1996). Further, at any time, an inactive firm can either wait to invest or invest in one of two types of assets that differ only on their opportunity costs of usage once in operation. In turn, a firm active in the market can either remain operative or disinvest and thus get the redeployment/resale value of its asset.

In deciding when and how to invest in the first place, firms must take into account that the choice of assets has two important implications. First, different types of resources shield differently against uncertainty, and hence have a different impact on preemptive incentives.[3] Second, a first entrant must also consider that the degree of resource specificity has a strategic effect on the behavior of the second entrant, in terms of both asset choice and entry timing. Thus, a preemptor should avoid choosing an asset that

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[2] See e.g. Vives (1989), Roller and Tombak (1990), Boyer and Moreaux (1997) or, more recently, Goyal and Netessine (2004).

[3] The benefit of moving first is enjoying (an expectation of) monopoly rents for some time or forever at the expense of foregoing the opportunity of entering second and thus enjoying the full benefit of informational flexibility, since a second-mover does not suffer from competition to enter in the last place

would not provide a credible commitment to remain active and would thereby allow its competitor to force it out at some later date even if the market is expanding. At least, its rival should have no incentives to force it out of a growing market if it can choose an asset to do so. Even if the follower could not (or did not wish to) force the preemptor out, the first-entrant's resource choice would have a strategic effect on the second-entrant's decision about which asset to acquire, as well as its entry timing. Both firms foresee that the degree of specificity of their assets determines their exit order –and, hence, their rents– once the market starts its irreversible decay. If both firms choose distinct resources, then the one with a specialized asset can be shown to outlast its rival, and thus will enjoy temporary monopoly profits at the expense of getting a less valuable outside option at a later time (which will also matter due to discounting). If both firms choose the same factors, then they will engage in a harsh competition to determine which firm exists first. Such war of attrition will fully dissipate all rents, which creates a tension to avoid it.

From a technical standpoint, this paper also makes a contribution to the literature on games of timing by enlarging the action space of an inactive firm from two elements (e.g., "enter" and "do not enter") to three. In particular, we present a new class of stochastic timing games in which two identical firms have to decide their timing of market entry as well as their entry mode, thus extending Fudenberg and Tirole's (1985) classic paper on preemptive incentives, since we focus on closed-loop strategies. Besides the binary entry mode, market exit is also allowed, unlike their framework.

Regarding the main results of the paper, we find that a second-entrant generally invests in a flexible resource, and firms usually end up investing in different types of assets despite they are ex ante identical. Another remarkable aspect is that, in spite of (potentially) asymmetric equilibrium configurations, neither firm expects to gain a higher equilibrium payoff. When deciding which firm preempts the other, the rent equalization principle (Fudenberg and Tirole 1985) applies across two distinct dimensions since firms can undertake a preemptive action in two distinct ways, due to the binary entry mode. However, firms need not attain the highest (equalized) rent due to the existence of two incentive compatibility constraints. In the first place, each firm may have incentives to let the rival enter early and force it out of an expanding market at a later date, after enjoying the informational benefit of waiting to invest for a while. Of course, the rival would foresee its premature exit and thus would not be willing to enter in the equilibrium outcome that would result in the highest rent for each. Hence, the violation of one of the incentive compatibility constraints would happen because firms cannot commit not to force the rival out of the market if they have the means (i.e., type of resource) to do so. In the second place, firms may have incentives to "tacitly" coordinate their actions so as to get the highest (equalized) rent. Yet, the first-entrant may be willing to break such non-binding agreement by choosing a different asset from the one it is supposed to choose at the date at which it has to enter. In this sense, the other incentive compatibility constraint may be violated due to the lack of incentives to sustain a collusive agreement.

The related literature of timing games to which our paper contributes has been particularly useful to study market entry and exit, as in the influential papers by Fudenberg and Tirole (1985) and Ghemawat and Nalebuff (1985), respectively. The rapidly growing game-theoretic real options literature has recently extended such settings to environments characterized by uncertainty that evolves over time (e.g., Murto 2004). Although our paper allows firms to exit after entering the market, it is not usual to find applications of timing games that consider entry and exit decisions jointly. As a result, the literature on strategic (dis)investment patterns generally neglects the interplay of incentives to undertake a preemptive move when entering a market and the commitment to outlast a rival if necessary. The only exceptions up to date are Londregan (1990) in a deterministic setting and Ruiz-Aliseda (2003) in a stochastic framework, so our paper constitutes an extension of this stream of the timing game literature using the Bayesian real options setup developed in Gutiérrez and Ruiz-Aliseda (2003).

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 examines the entry and exit patterns in a declining market, while Section 4 does the same when the market is in expansion. Section 5 concludes by discussing other interpretations of the model developed in previous sections.

2. THEORETICAL MODEL

Time, labeled *t*, is continuous on $[0,\infty)$, and there are two firms that are initially out of a market that grows until a random date $\tilde{\tau}$, and declines thereafter. The market maturity date is exponentially distributed with parameter $\lambda > 0$ and the density is denoted by

$h(\tau)$. If there are $n \in \{1, 2\}$ firms active in the industry, then, for a given realization τ of $\widetilde{\tau}$, the temporal evolution of flow profits is as follows:

$$\Pi_n(t, \tau) = \begin{cases} G_n(t) & \text{if } t < \tau \\ D_n(t - \tau) & \text{if } t \geq \tau \end{cases}$$

We assume that $G_1(\cdot) > G_2(\cdot)$, $D_1(\cdot) > D_2(\cdot)$, $G_2(0) > 0$ and $D_2(0) > 0$. In addition, $G_n(\cdot)$ and $D_n(\cdot)$ are assumed to be continuously differentiable functions for all $n = 1, 2$, with positive and negative slopes, respectively. Furthermore, we let $\lim_{t \to \infty} D_n(t - \tau) = 0 \; (n = 1, 2)$ so that firms exit the market due only to the existence of opportunity costs of operation. Lastly, we rule out corner solutions by assuming that $G_1(0) \leq D_1(0)$, and we also let both $G_1(\cdot)$ and $G_2(\cdot)$ be unbounded, with the following restriction: $\int_0^\infty G_1(t) e^{-(r+\lambda)t} dt < \infty$. This bounds the expected payoffs of firms.

In general, there is a discontinuous jump in the sample path of flow profits so that each firm's continuation payoff past the maturity date is non-contingent on the realization of $\widetilde{\tau}$. Yet, it is worth noting that the date at which such continuation payoff is attained is random, which creates uncertainty about the payoff associated to the decline stage of the market. The discontinuity allows us to simplify the analysis of investment decisions while the market is expanding, the main focus of the paper, and allows us to make assumptions that rule out entry into a decaying market (an empirically irrelevant phenomenon). If the random pattern of market growth and decline were continuous, it seems reasonable that entry would not take place in a declining market either. Otherwise, why would a firm not enter while the market had growth prospects (e.g., a little bit earlier than the maturity date), but would enter once it is known it is going to die?

Firms are also assumed to be risk-neutral and to discount payoffs at the rate $r > 0$. Any firm can enter the market with a non-specific asset or a specialized one by incurring an investment cost $K \geq \dfrac{D_1(0)}{r}$. Such cost is large enough so as to prevent entry into a declining market even in a monopoly situation, and is consistent with the assumption that at most two firms are willing to enter the market. A firm with a general-purpose factor can exit and get the (one-time) resale/redeployment value of its asset, $R_g < \frac{D_2(0)}{r}$.[4] In turn, the opportunity cost of a specialized resource is $R_s \in (0, R_g)$.

Given that the main focus of the paper is on the (dis)advantages of resource flexibility when entering a new market whose evolution is uncertain, we do not allow firms to enter the market after exiting, which would formidably complicate the model. Firms are assumed to use feedback strategies and we will focus on the symmetric Markov Perfect Equilibrium (MPE) of the game.[5] Despite the lack of firm-specific uncertainty, symmetric MPE outcomes exhibit the interesting property that even ex ante identical firms end up being different ex post, at least regarding their entry timing, and sometimes their asset choice.

Finally, we follow Dutta, Lach and Rustichini (1995) and assume that if both firms attempt to enter at some date *t*, then only one of them succeeds in doing so. In particular, the probability of entry by any firm is one-half. In Fudenberg and Tirole (1985), such probability is derived endogenously, and it can be shown that in our setting it must be equal to one-half, at least in the relevant cases. We omit the details given their accessory character to the paper.

Before proceeding to solving the model, it is worth noting that in our real options setup, firms can partly hedge against adverse realizations of uncertainty in a variety of ways. More precisely, this can be done by waiting to invest (informational flexibility) and/or recouping a large fraction of the investment cost (resource flexibility). Actually, both aspects interact with each other, and, intuitively, a smaller resource flexibility induces a firm to benefit more from informational flexibility, at least in a non-competitive setting. As will become clearer in Section 4, this need not be true in our strategic competition setup because of the strategic effects of asset specificity.

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[4] This assumption can be easily relaxed to make R_g smaller than K. The same analysis goes through without any substantial changes (see Ruiz-Aliseda 2003).

[5] This makes past history irrelevant, except for the current level of payoff-relevant state variables. The state at time *t* consists of the current date, the identity of the firm/s active at such date and their respective assets, and the period of time gone by since the realization of the market maturity became known to the firms (with 0 denoting the fact that firms still do not know what the maturity date is).

3. DECLINING MARKET: ENTRY AND EXIT PATTERNS

It is clear that in our simple setup, no firm is willing to enter a declining market, no matter if any of them is active or not, since $K > \frac{D_1(0)}{r}$, which implies that even a monopolist entering right after the maturity date would not cover its investment cost.[6] Let us call a firm with a general-purpose (specialized) asset a type-*g* (type-*s*) firm. Also, let t_q^u denote the equilibrium exit date of a type-*q* firm, $q \in \{g, s\}$, when it competes in a declining market with a type-*u* rival, $u \in \{\phi, g, s\}$, where ϕ is a convention used to denote the absence of an active competitor. Given that no firm is willing to enter a dying market, it is clear that, in equilibrium, a firm of type $q \in \{g, s\}$ which finds itself alone in a declining market exits at

$$t_q^{\phi}(\tau) = \arg\max_{t \geq \tau} \int_{\tau}^{t} D_1(x-\tau) e^{-r(x-\tau)} dx + R_q e^{-r(t-\tau)} = \tau + D_1^{-1}(rR_q).$$

However, if both coincide in a declining market, there are two different kinds of cases to consider. On the one hand, if both have different types of assets, the unique MPE outcome is characterized by the firm with the non-specific resource exiting first at

$$t_g^{s}(\tau) = \arg\max_{t \geq \tau} \int_{\tau}^{t} D_2(x-\tau) e^{-r(x-\tau)} dx + R_g e^{-r(t-\tau)} = \tau + D_2^{-1}(rR_g).$$

In turn, the firm with the specialized asset exits later at

$$t_s^{g}(\tau) = \arg\max_{t \geq \tau} \int_{\tau}^{t_g^s(\tau)} D_2(x-\tau) e^{-r(x-\tau)} dx + \int_{t_g^s(\tau)}^{t} D_1(x-\tau) e^{-r(x-\tau)} dx + R_s e^{-r(t-\tau)} = \tau + D_1^{-1}(rR_s).$$

The proof of this result can be found in Ruiz-Aliseda (2003, Proposition 1), although the intuition for such an MPE outcome is straightforward. A type-*g* firm has a higher tension to exit, given its higher redeployment/resale value, so its lower exit barriers make it exit earlier. This outcome is supported by the credible threat made by the competitor of triggering a war of attrition, as in Ghemawat and Nalebuff (1985). As a result, the firm with a specialized factor can safely exit at $t_s^g(\tau) = \tau + D_1^{-1}(rR_s) = t_s^{\phi}(\tau)$, i.e., its optimal exit date in monopoly.

[6] We omit these details and refer the interested reader to Proposition 2 in Ruiz-Aliseda (2003).

On the other hand, the second case to consider is that in which both firms have the same kind of asset, type $q \in \{g, s\}$ say. As mentioned earlier, we restrict our attention to symmetric MPE. There exists a unique symmetric equilibrium, and it involves mixed strategies. Letting $t_q^q(\tau) = \arg\max_{t \geq \tau} \int_\tau^t D_2(x-\tau)e^{-r(x-\tau)}dx + R_q e^{-r(t-\tau)} = \tau + D_2^{-1}(rR_q),$ it can be shown that, given a known maturity date τ, each firm exits before time $t \in [t_q^q(\tau), t_q^\phi(\tau)]$ according to a certain cumulative distribution function $J_q(t,\tau) = 1 - \exp(-\int_{t_q^q(\tau)}^t \mu_q(x,\tau)dx),$ where $\mu_q(t,\tau) = \dfrac{rR_q - D_2(t-\tau)}{e^{r(t-\tau)} \int_{t-\tau}^{t_q^\phi(\tau)-\tau} (D_1(x) - rR_q)e^{-rx}dx},$ as long as the rival firm has not exited.[7] Of course, any firm stays until $t_q^\phi(\tau)$ if the competitor exits before $t_q^\phi(\tau)$. Given the nature of such mixed strategy equilibrium, each firm expects to gain a continuation payoff of $\int_\tau^{t_q^q(\tau)} D_2(x-\tau)e^{-r(x-\tau)}dx + R_q e^{-r(t_q^q(\tau)-\tau)}$ $(q \in \{g,s\})$ at the maturity date τ. Henceforth, let C_q^u $(q,u \in \{g,s\})$ denote the continuation payoff of a type-*q* firm discounted back to date τ for the subgames in which the market is declining and it is competing head-to-head with a type-*u* rival. Making a straightforward change of variables, we can summarize all the results of this section as follows:

[7] Although we will not go into details, standard arguments show that $J_q(\cdot)$ has no atoms and its support is the connected set $[t_q^q(\tau), t_q^\phi(\tau)]$. Since $J_q(\cdot)$ has no atoms, the Nash equilibrium is subgame perfect. Intuitively, $\mu_q(\cdot)$ is such that each firm is indifferent between exiting and staying for any $t \in [t_q^q(\tau), t_q^\phi(\tau)]$ (note that this makes use of the fact that, if both firms stay until $t_q^\phi(\tau)$, then none of them ends up attaining the "prize" of a monopoly position, whereas the cost of delaying exit until $t_q^\phi(\tau)$ is indeed positive). So $\mu_q(\cdot)$ is such that the marginal value of remaining one more infinitesimal unit of time in the market, $D_2(t-\tau)dt + (\int_t^{t_q^\phi} (D_1(x-\tau) - rR_q)e^{-r(x-t)}dx)\mu_q(t,\tau)dt,$ equals the marginal cost of waiting to exit, $rR_q dt.$ By delaying exit, a type-*q* firm earns the duopoly profit and acquires an option to remain alone in the market until the optimal monopoly exit date, at the expense of foregoing R_q for some time.

$$C_q^{\phi} = \int_{\tau}^{t_q^{\phi}(\tau)} D_1(x-\tau)e^{-r(x-\tau)}dx + R_q e^{-r(t_q^{\phi}(\tau)-\tau)} = \int_0^{D_1^{-1}(rR_q)} D_1(t)e^{-rt}dt + R_q e^{-rD_1^{-1}(rR_q)}, q \in \{g,s\};$$

$$C_q^{q} = \int_{\tau}^{t_q^{q}(\tau)} D_2(x-\tau)e^{-r(x-\tau)}dx + R_q e^{-r(t_q^{q}(\tau)-\tau)} = \int_0^{D_2^{-1}(rR_q)} D_2(t)e^{-rt}dt + R_q e^{-rD_2^{-1}(rR_q)}, q \in \{g,s\};$$

$$C_g^{s} = \int_{\tau}^{t_g^{s}(\tau)} D_2(x-\tau)e^{-r(x-\tau)}dx + R_g e^{-r(t_g^{s}(\tau)-\tau)} = \int_0^{D_2^{-1}(rR_g)} D_2(t)e^{-rt}dt + R_g e^{-rD_2^{-1}(rR_g)}; \text{ and}$$

$$\begin{aligned} C_s^{g} &= \int_{\tau}^{t_g^{s}(\tau)} D_2(x-\tau)e^{-r(x-\tau)}dx + \int_{t_g^{s}(\tau)}^{t_s^{g}(\tau)} D_1(x-\tau)e^{-r(x-\tau)}dx + R_s e^{-r(t_s^{g}(\tau)-\tau)} \\ &= \int_0^{D_2^{-1}(rR_g)} D_2(t)e^{-rt}dt + \int_{D_2^{-1}(rR_g)}^{D_1^{-1}(rR_s)} D_1(t)e^{-rt}dt + R_s e^{-rD_1^{-1}(rR_s)}. \end{aligned}$$

Finally, notice that the facts that $\frac{dC_g^s}{dR_g} > 0$ and $t_g^g(\tau) = t_g^s(\tau)$ imply that $C_s^s < C_g^g = C_g^s$, while the comparison between C_g^s and C_s^g is generally ambiguous, which turns out to be crucial in the analysis below. Intuitively, $C_s^g \geq C_g^s$ if and only if the profit stream made by a firm in monopoly is sufficiently high relative to the difference in redeployment values and the difference in the equilibrium exit times.

4. GROWING MARKET: ENTRY AND EXIT PATTERNS

Let us now focus on subgames in which the maturity of the market is unknown to the firms, taking into account the results of the previous section. To identify the symmetric MPE outcome, we follow three steps. First, we study the expected payoff of a firm that enters in the second place as a function of its investment time and mode of entry, assuming that it takes its rival's entry in the first place as a *fait accompli* and thus does not attempt to force the incumbent out. In this sense, the roles as a leader or as a follower are assumed to be exogenously preassigned. Second, we examine the payoff of an "exogenous leader" as a function of its entry time and its asset choice, accounting for the follower's optimal best-response, maintaining the assumption that the follower does not try to induce the leader's exit. Third and last, we endogenize the identities of the leader and follower by dropping the previous temporary assumptions so as to account for the incentives to undertake a preemptive investment and to force a rival out of the market, which yields the equilibrium outcome of the entire game.

We distinguish two exclusive cases throughout. In the first place, we let $C_g^s > C_s^g$, i.e., the firm with a general-purpose asset gains a higher payoff than the firm that owns a specialized one when both coincide during the market decline. As explained above, we proceed to construct the follower's payoff function for each type of resource that the leader may have chosen. In particular, if the leader is of type $u \in \{g, s\}$ and the follower invests in an asset of kind $q \in \{g, s\}$ at date t, then the latter expects to gain:

$$F_q^u(t) = \int_t^\infty h(\tau)\left(\int_t^\tau G_2(x)e^{-rx}dx + C_q^u e^{-r\tau} - Ke^{-rt}\right)d\tau.$$

Thus, if the realized maturity date is smaller than the entry time chosen by the follower, then it gains nothing (since it does not invest in a dying market). However, if the maturity date allows for its entry, then it pays the discounted entry cost in order to gain a stream of discounted duopoly profits while the market is growing, and the continuation payoff associated to a declining market given the types of assets chosen by the two firms.

Since $C_g^g = C_g^s > C_s^g$ and $C_g^s > C_s^s$, it is straightforward to see that the follower best-responds by investing in a non-specific asset at date $f_g^u = \arg\max_t F_g^u(t) = G_2^{-1}((r+\lambda)K - \lambda C_g^u)\ \forall\, u \in \{g, s\}$.[8] Indeed, the fact that $C_g^s = C_g^g$ implies that $f_g^s = f_g^g$.

We can now analyze the payoff function of a leader given the best-response of the rival firm as a follower, taking into account that no type of follower has incentives to exit an expanding market if it enters at or later than $f_g^s = f_g^g$,[9] and no firm is willing to exit while in monopoly, as shown by the following proposition:

Proposition 1: In a Markov Perfect Equilibrium, no firm ever exits while alone in a growing market.

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[8] Since $F_q^u(t)$ can be easily shown to be strictly quasi-concave for all $q, u \in \{g, s\}$.

[9] This follows because: (*i*) a type-*q* firm which competes against a type-*u* firm is not willing to exit a growing market for $t \geq x_q^u$ (as shown in the proof of Proposition 2 below, in which x_q^u is defined) and; (*ii*) the increasingness of $G_2(\cdot)$ implies that $x_q^u < f_q^u\ \forall\, q, u \in \{g, s\}$.

Proof: See Appendix.■

Therefore, we have that if one of the firms enters first with an asset of type $q \in \{g, s\}$ at date *t*, the fact that its competitor best-responds by entering with a general-purpose asset at $\max(t, f_g^q)$ implies that the leader's expected payoff as a function of its entry date is:

$$L_q^g(t) = \begin{cases} \int_t^{f_g^q} h(\tau)\left(\int_t^{\tau} G_1(x)e^{-rx}dx + C_q^{\phi}e^{-r\tau} - Ke^{-rt}\right)d\tau + \\ \int_{f_g^q}^{\infty} h(\tau)\left(\int_t^{f_g^q} G_1(x)e^{-rx}dx + \int_{f_g^q}^{\tau} G_2(x)e^{-rx}dx + C_q^g e^{-r\tau} - Ke^{-rt}\right)d\tau & \text{if } t \le f_g^q \\ \int_{f_g^q}^{\infty} h(\tau)\left(\int_{f_g^q}^{\tau} G_2(x)e^{-rx}dx + C_q^g e^{-r\tau} - Ke^{-rt}\right)d\tau & \text{if } t > f_g^q \end{cases}$$

Letting $l_q^g = \arg\max_t L_q^g(t)$, $q \in \{g, s\}$, and assuming henceforth that $G_1^{-1}((r+\lambda)K - \lambda C_s^{\phi}) < G_2^{-1}((r+\lambda)K - \lambda C_g^s)$,[10] it is easy to see that the following holds:

$$l_g^g = G_1^{-1}((r+\lambda)K - \lambda C_g^{\phi}) < l_s^g = G_1^{-1}((r+\lambda)K - \lambda C_s^{\phi}).^{11}$$

Intuitively, the leader believes that entry by the follower would destroy part of its monopoly profits with a certain probability, and thus the follower's subsequent investment would not have an effect on the marginal payoff of the leader. Therefore, marginal differences between leading with a flexible asset and a specialized one stem only from differences in the continuation payoff associated to a declining market that is monopolized by the leader. In consequence, the result is entirely driven by the well-known fact that a general-purpose asset is more profitable than a specialized one when the market is declining (since $\frac{dC_g^{\phi}}{dR_g} > 0$, so $C_g^{\phi} > C_s^{\phi}$).

[10] We ignore the situations in which $G_1^{-1}((r+\lambda)K - \lambda C_s^{\phi}) \ge G_2^{-1}((r+\lambda)K - \lambda C_g^s)$, because the implications are trivial, since no firm has incentives to enter in the first place with a specialized asset. The reason is that, in that case, $L_s^g(t) < F_g^s(f_g^s)\ \forall t$, given that $l_s^g = G_2^{-1}((r+\lambda)K - \lambda C_s^g)$. The MPE outcome is thus characterized by one of the firms entering first with a general-purpose asset and the other investing in the same type of resource as well, but at a later time. See Proposition 3 for more details about the features of this type of equilibrium.

[11] See Ruiz-Aliseda (2003) for a proof of the single-peakness of the leader's payoff function.

In addition, the facts that $f_g^g = f_g^s$, $C_g^g = C_g^s > C_s^g$ and $C_g^\phi > C_s^\phi$ imply that $L_g^g(t) > L_s^g(t)\,\forall\, t$, so it follows that a first-entrant that cannot be forced out always prefers to lead with a non-specific asset. Of course, this need not be true when entry by the second-mover would induce the first-entrant to exit the market. Indeed, a follower has so far been assumed to accept its competitor's entry as a *fait accompli* (i.e., it does not try to induce its rival's exit by speeding up entry). It has also been assumed that it accepts its follower role (i.e., it does not try to preempt its rival). We now drop these two restrictions and pay attention to each separately, and we then analyze how they interact so as to solve the entire game.

To examine incentives to force a rival out of the market, consider subgames in which both firms are active in a growing market, and suppose each has a different type of asset.[12] Let c_q^u denote the earliest date at which a type-*q* firm prefers staying in a duopolistic industry rather than getting its redeployment/resale value when competing against a type-*u* rival, with $q, u \in \{g, s\}$ and $q \neq u$.[13] Recalling that reentry is not allowed, its formal definition is as follows:

$$c_q^u = \inf\left\{ t \in [0,\infty) : R_q e^{-rt} \leq \int_t^\infty \frac{h(\tau)}{\Pr(t < \tilde{\tau})} \left(\int_t^\tau G_2(x) e^{-rx} dx + C_q^u e^{-r\tau} \right) d\tau \right\}.$$

Because $C_g^s > C_s^g$, we can have either $c_s^g < c_g^s$ or $c_g^s < c_s^g$,[14] which can be shown to imply the following:

Proposition 2: Assume that at time *t* both firms were active in an expanding market. Then, for $t \leq \max(c_s^g, c_g^s)$, the type-*s* firm would exit at date *t* and its competitor would stay if $c_g^s < c_s^g$, while the type-*g* firm would exit at *t* and its rival would remain active if $c_s^g < c_g^s$. Both firms would remain operative in the market for $t > \max(c_s^g, c_g^s)$.

[12] If each firm plays a symmetric (mixed-strategy) equilibrium when they have the same type of asset, then both are indifferent between exiting immediately and staying a little bit more at any date at which there is no enough room in the industry for the two of them. (In this case, the probability distribution according to which they mix need not be atomless, though.) Therefore, a potential entrant would prefer to wait and see rather than engage in a war of attrition from which it expects to gain nothing.

[13] The proof of Proposition 2 shows that such date exists and is unique.

[14] To simplify matters that do not add any insight to the analysis, we impose a very weak restriction on the parameter space and assume that it is such that $c_g^s \neq c_s^g$.

Proof: See Appendix.■

Using Proposition 2, we can compute the expected payoff of a firm that foresees forcing its rival out of the market when it enters at time t with a type-q asset:

$$L_q^{\phi}(t) = \int_t^{\infty} h(\tau) \left(\int_t^{\tau} G_1(x) e^{-rx} dx + C_q^{\phi} e^{-r\tau} - K e^{-rt} \right) d\tau.$$

The fact that future entry is perceived as having a fixed effect on a leader's entry decision implies that, for $t \le f_u^q$, $L_q^{\phi}(t)$ is equal to $L_q^u(t)$ plus a positive constant. More precisely:

$$L_q^{\phi}(t) = L_q^u(t) + \int_{f_u^q}^{\infty} h(\tau) \left(\int_{f_u^q}^{\tau} (G_1(x) - G_2(x)) e^{-rx} dx + (C_q^{\phi} - C_q^u) e^{-r\tau} \right) d\tau.$$

However, before studying the implications of the credibility to remain in a duopolistic market and the two cases that we discussed are possible, let us proceed to the last step of the analysis, which requires considering preemptive incentives. In particular, define $p_q^u = \inf\{t \ge 0 : L_q^u(t) \ge F_u^q(f_u^q)\}$, $q,u \in \{g,s\}$, and note that a firm that plans to acquire a type-q factor and foresees that its rival will be of type-u has no incentives to enter before p_q^u, since $L_q^u(t) < F_u^q(f_u^q)\ \forall\, t < p_q^u$. To finalize the analysis, it only remains to perform a joint examination of the incentives to preempt the rival and to force it out the market if it is already active, recalling that two exclusive cases are plausible: $c_s^g < c_g^s$ or $c_g^s < c_s^g$.

In the first place, if $c_g^s < c_s^g$, a firm that enters with a flexible asset is shielded against attempts to force it out when the market is expanding. This, together with the fact that $L_g^g(t) > L_s^g(t)$, implies that any first-entrant must invest in a general-purpose asset because it cannot change the follower's subsequent action by investing in a specialized factor, and in addition it gains a higher continuation payoff in a declining market both in monopoly and duopoly. As a result, both firms acquire a non-specific resource, because the stream of monopoly profits to be reaped after the rival has exited is not large enough so as to make the follower choose a specialized asset. By the rent equalization principle (REP) (see Fudenberg and Tirole 1985), the leader and the follower must attain the same equilibrium expected payoff, so the firm that invests first ends up entering at p_g^g, and the second-mover ends up entering at f_g^g. This situation is represented in Figure 1.

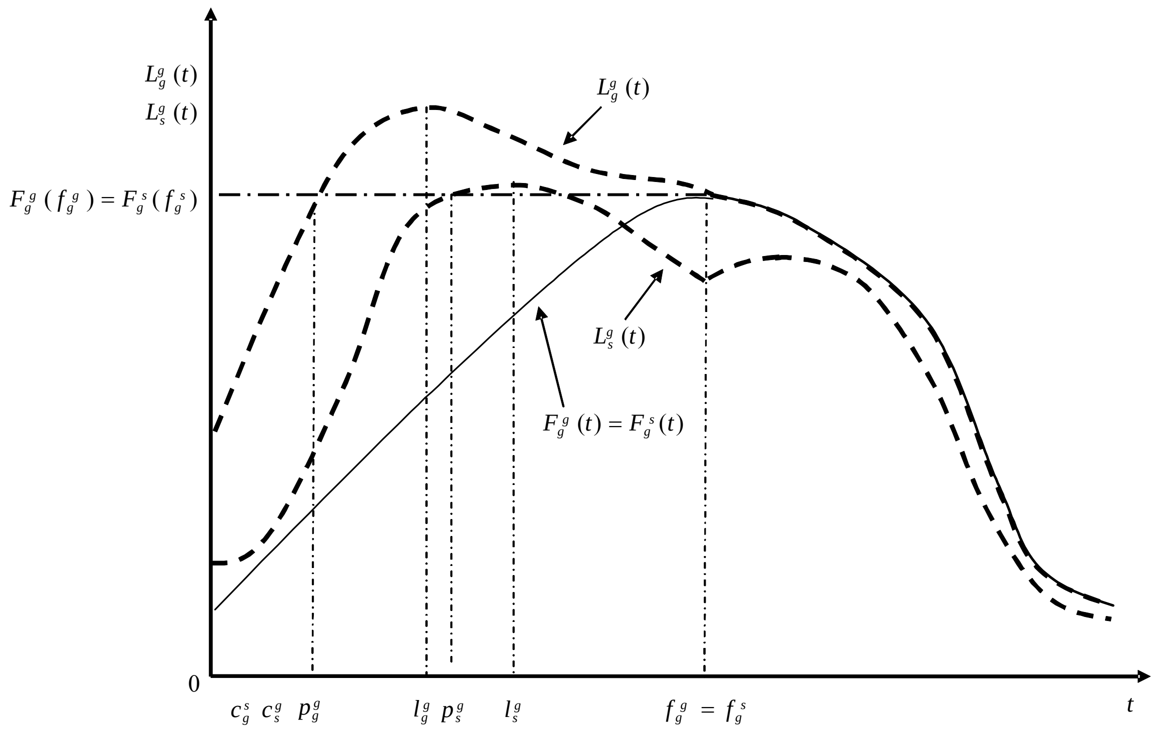

Figure 1

In the second place, if $c_s^g < c_g^s$, a firm that enters with a flexible asset can be induced to exit if its rival enters no later than c_g^s. Recalling that any firm has a guaranteed payoff of $F_g^g(f_g^g)$ by not engaging in preemptive play and adopting a follower's role, let us distinguish two subcases. If $c_g^s \leq p_g^g$, then the fact that $L_g^g(t) > L_s^g(t)$, and the increasingness of $L_g^g(\cdot)$ at dates prior to p_g^g imply the following for all $t \leq c_g^s$:

$$L_s^g(t) < L_g^g(t) \leq L_g^g(c_g^s) \leq L_g^g(p_g^g) = F_g^g(f_g^g).$$

As a result, no firm has incentives to undertake a preemptive move by investing in a specialized factor on the set of dates $[0, c_g^s]$. Therefore, the preemptor must enter with a general-purpose asset, and the same equilibrium outcome as when $c_g^s < c_s^g$ follows.

It only remains to study the cases in which both $p_g^g < c_g^s$ and $c_s^g < c_g^s$. It is clear that, if the preemptor invested in a general-purpose factor before c_g^s, then it would be forced out by its rival at $\min(l_s^g, c_g^s)$ if and only if $L_s^{\phi}(\min(l_s^g, c_g^s)) > F_g^g(f_g^g)$. So when $L_s^{\phi}(\min(l_s^g, c_g^s)) \leq F_g^g(f_g^g)$, a second-mover would have no incentives to force the incumbent out, even though it would have the means to do so, and the REP would imply that the first-entrant could safely invest in a flexible resource at $p_g^g < c_g^s$, while its rival would enter at f_g^g with the same type of asset. However, if $L_s^{\phi}(\min(l_s^g, c_g^s)) > F_g^g(f_g^g)$, the fear of being forced out if the market grows enough would imply that no firm would be willing to enter first with a flexible asset at p_g^g. As can be seen in Figure 2, the REP would imply that the first-mover should enter with a specialized factor at p_s^g,[15] while the second-mover should invest in a non-specific asset at $f_g^s = f_g^g$.

There is one more necessary condition for the existence of a MPE, though. At time p_s^g, the first-entrant should have no incentives to deviate and invest in a general-purpose asset, which would give a higher payoff $L_g^g(p_s^g) > L_s^g(p_s^g)$ if it were not forced out later on. More specifically, if $c_g^s \leq p_s^g$, a first-entrant that deviated and invested in a flexible factor could not be induced to exit. As a result, we have shown that no symmetric MPE exists if the following two conditions were met: $L_s^{\phi}(\min(l_s^g, c_g^s)) > F_g^g(f_g^g)$ and $\max(p_g^g, c_s^g) < c_g^s \leq p_s^g$. However, if $p_s^g < c_g^s$ and a first-entrant invested in a flexible resource at p_s^g, then such firm would be forced out of the market by its rival, which would enter with a specialized asset at $\min(l_s^g, c_g^s)$ because it would yield a greater payoff than entering with a general-purpose asset at f_g^g.[16] In this type of MPE, entry using a specific resource to preempt the rival would take place at a later date than when a flexible asset is used (i.e., $p_g^g < p_s^g$), as $F_g^s(f_g^s) = F_g^g(f_g^g) = L_g^g(p_g^g) > L_s^g(p_g^g)$. Yet, the

[15] This makes use of the fact that $L_s^g(\min(l_s^g, c_g^s)) > F_g^g(f_g^g) = F_g^s(f_g^s) = L_s^g(p_s^g)$.

[16] Recall that $L_s^{\phi}(\min(l_s^g, c_g^s)) > F_g^g(f_g^g)$.

equilibrium outcome in the latter case we have analyzed would be supported by a credible threat of forcing the rival out of the market if it entered first at p_g^g with a non-specific factor. As a result, entry by the first-mover would be delayed, even though it would finally end up gaining the same expected payoff, since $L_g^g(p_g^g) = F_g^g(f_g^g) = F_g^s(f_g^s) = L_s^g(p_s^g)$.

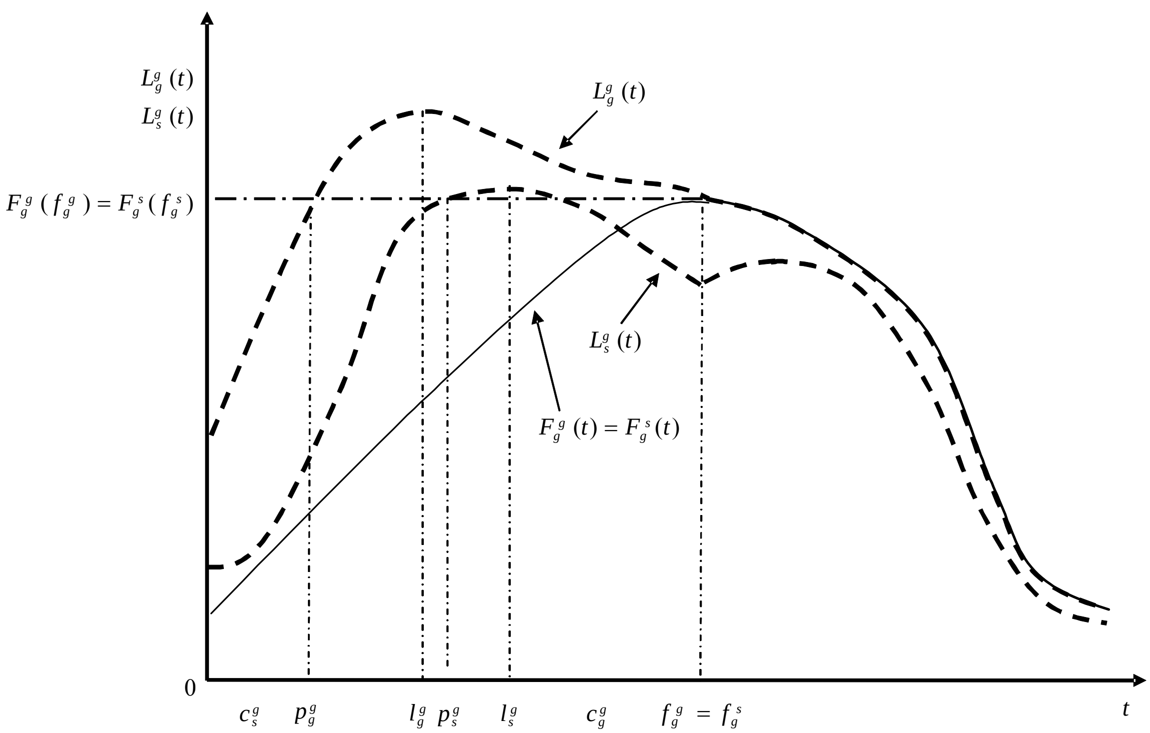

Figure 2

We can now put together this wide array of results and summarize what we have shown for the cases in which the firm that operates a flexible asset gains a higher payoff than the firm with a specialized one throughout the decline of a duopolistic industry:

Proposition 3: Let $C_g^s > C_s^g$, and distinguish three cases:

(i) If $\max(p_g^g, c_s^g) \geq c_g^s$ and/or $L_s^\phi(\min(l_s^g, c_g^s)) \leq F_g^g(f_g^g)$, then, with probability one-half, one of the firms enters with a general-purpose asset at p_g^g and the other enters with a general-purpose asset at f_g^g, while with probability one-half the identities of

the firms are interchanged. Each attains the same equilibrium payoff: $L_g^g(p_g^g) = F_g^g(f_g^g)$.

(ii) If $\max(c_s^g, \max_{q\in\{g,s\}} p_q^g) < c_g^s$ and $L_s^\phi(\min(l_s^g, c_g^s)) > F_g^g(f_g^g)$, then, with probability one-half, one of the firms enters with a specialized asset at p_s^g and the other enters with a general-purpose asset at f_g^s, while with probability one-half the identities of the firms are interchanged. Each attains the same equilibrium payoff: $L_s^g(p_s^g) = F_g^s(f_g^s)$.

*(iii)*If $L_s^\phi(\min(l_s^g, c_g^s)) > F_g^g(f_g^g)$ and $\max(p_g^g, c_s^g) < c_g^s \le p_s^g$, then no symmetric MPE exists.

Let us refer to the MPE in which the first entrant enters with a type-*q* asset and the second entrant invests in a type-*u* resource as a *q*-*u* equilibrium, $q,u \in \{g,s\}$. Then, straightforward comparative statics show the following:

Proposition 4: In a *g*-*g* MPE, as R_g increases, both firms' equilibrium payoff increases, while the follower speeds up its entry time. In an *s*-*g* MPE, an increase in R_s affects neither equilibrium payoffs nor the second-mover's entry time, while the first-entrant speeds up its investment in the specific asset. A higher R_g increases the payoff attained by both firms in equilibrium, and speeds up the follower's entry time.

Proof: See Appendix.■

Intuitively, when R_g increases in a *g*-*g* equilibrium, the value of following with a general-purpose resource in an optimal fashion increases because the redeployment value of the asset is higher, which also implies that $\frac{df_g^g}{dR_g} < 0$. However, $\frac{dp_g^g}{dR_g}$ need not have a negative sign. The reason is that, on the one hand, a first-entrant foresees that its competitor will follow earlier, so the market structure will switch from monopoly to duopoly at an earlier date, which is negative from the preemptor's viewpoint. On the other hand, the first-mover benefits from an increase in R_g because its position when exiting is improved if the market does not grow so much so as to allow the rival's entry.

The net effect of these counteracting forces is ambiguous, and thus $\frac{dp_g^g}{dR_g}$ cannot be signed in general.[17]

As for an *s*-*g* equilibrium, the value of being a type-*s* first-mover in a declining market increases by the same amount if R_s increases, no matter if the market structure at the maturity date is monopolistic or duopolistic. The reason is of course that a firm with a specific asset always exits at its optimal monopoly date. So the facts that the value of preemption increases and the value of entering as a second-mover in an optimal way does not vary imply that firms fight more aggressively to become the first-entrant, which speeds up the date at which preemption takes place. In turn, augmenting R_g increases the value of entering in the second place with a general-purpose asset. However, a higher R_g speeds up both entry and exit by the second-entrant, and thus the effect on the value of using a specialized resource to preempt a rival firm is ambiguous.

From now on, we focus on the cases in which the continuation payoff of a type-*s* firm is (weakly) higher than that of a type-*g* firm when both coincide in a declining market (i.e., $C_g^s \le C_s^g$). As for the follower's payoff function, it is as follows when it enters with a type-*q* plant and competes against a type-*u* plant:

$$F_q^u(t) = \int_t^\infty h(\tau)\left(\int_t^\tau G_2(x)e^{-rx}dx + C_q^u e^{-r\tau} - Ke^{-rt}\right)d\tau.$$

[17] In equilibrium, we have that $L_g^g(p_g^g(R_g), R_g) = F_g^g(f_g^g(R_g), R_g)$, totally differentiating with respect to R_g (with the aid of the envelope theorem), canceling terms and rearranging yields:

$$\frac{dp_g^g}{dR_g} = \frac{[(G_1(f_g^g) + \lambda C_g^\phi - (r+\lambda)K)e^{-(r+\lambda)f_g^g}]\frac{df_g^g}{dR_g} + \int_{p_g^g}^{f_g^g} h(\tau)e^{-r\tau}\left(\frac{dC_g^\phi}{dR_g}\right)d\tau}{(G_1(p_g^g) + \lambda C_g^\phi - (r+\lambda)K)e^{-(r+\lambda)p_g^g}}.$$

Given that $G_1(f_g^g) + \lambda C_g^\phi - (r+\lambda)K = G_1(f_g^g) - G_2(f_g^g) + \lambda(C_g^\phi - C_g^g) > 0$, $\frac{df_g^g}{dR_g} < 0$, $\frac{dC_g^\phi}{dR_g} > 0$, and $G_1(p_g^g) + \lambda C_g^\phi - (r+\lambda)K = G_1(p_g^g) - G_1(l_g^g) < 0$, it follows that $\frac{dp_g^g}{dR_g}$ cannot be given a sign in general because of the counteracting effect of the two forces on the numerator.

Since $C_s^g \geq C_g^s = C_g^g$ and $C_g^s > C_s^s$, a follower best-responds by investing in a different type of resource from that of its competitor. In particular, when the leader enters with an asset of type $u \in \{g,s\}$, a type-q follower enters at date $f_q^u = \arg\max_t F_q^u(t) = G_2^{-1}((r+\lambda)K - \lambda C_q^u),\ q \neq u.$ Since $C_s^g \geq C_g^s$, the follower enters later when the leader invests in a specialized asset: $f_s^g \leq f_g^s$.

Once we know the best-response of the rival firm when following, it is straightforward to construct the payoff function of a leader, recalling that no firm has incentives to exit a growing market while alone (by Proposition 1). If one of the firms invests in the first place in an asset of type $q \in \{g,s\}$, then the fact that its competitor best-responds by entering with the other type of asset denoted by $u \neq q$ implies the following:

$$L_q^u(t) = \begin{cases} \displaystyle\int_t^{f_u^q} h(\tau)\left(\int_t^{\tau} G_1(x)e^{-rx}dx + C_q^{\phi}e^{-r\tau} - Ke^{-rt}\right)d\tau + \\ \displaystyle\int_{f_u^q}^{\infty} h(\tau)\left(\int_t^{f_u^q} G_1(x)e^{-rx}dx + \int_{f_u^q}^{\tau} G_2(x)e^{-rx}dx + C_q^u e^{-r\tau} - Ke^{-rt}\right)d\tau & \text{if } t \leq f_u^q \\ \displaystyle\int_{f_u^q}^{\infty} h(\tau)\left(\int_{f_u^q}^{\tau} G_2(x)e^{-rx}dx + C_q^u e^{-r\tau} - Ke^{-rt}\right)d\tau & \text{if } t > f_u^q \end{cases}$$

Letting $l_q^u = \arg\max_t L_q^u(t) = G_1^{-1}((r+\lambda)K - \lambda C_q^{\phi}),\ \ q \in \{g,s\}$, it is easy to see that the following must hold: $l_g^s < l_s^g < f_s^g \leq f_g^s$.

To avoid a trivial outcome, we assume throughout that $L_g^s(l_g^s) \geq F_s^g(f_s^g)$.[18] Also, note that the facts that $f_s^g \leq f_g^s$, $C_s^g \geq C_g^s$ and $C_g^{\phi} > C_s^{\phi}$ imply that $\max(L_g^s(t), L_s^g(t))$ need not be equal to either $L_s^g(t)$ or $L_g^s(t)$ for all t, which slightly complicates the analysis relative to the cases in which $C_g^s > C_s^g$.

[18] Otherwise, $L_g^s(t) < F_s^g(f_s^g)\ \forall\, t$, and the equilibrium of the game would be characterized by one of the firms entering first with a specialized asset and the other with a general-purpose resource at a later time. See Proposition 5 for more details about this type of MPE.

As before, we now turn to analyzing the incentives to force a rival out of the market, so we consider subgames with both firms active in a growing market, although each is supposed to have a different type of resource. In this situation, we claim that $c_s^g < c_g^s$. To see this, first note that because $R_g > R_s$ and $C_s^g \geq C_g^s$, the following holds:

$$F_s^g(t) + (K - R_s)e^{-(r+\lambda)t} > F_g^s(t) + (K - R_g)e^{-(r+\lambda)t}.$$

So $F_s^g(t) + (K - R_s)e^{-(r+\lambda)t} > 0$ if $F_g^s(t) + (K - R_g)e^{-(r+\lambda)t} \geq 0,$ whence the desired result follows, since $F_q^u(t) + (K - R_q)e^{-(r+\lambda)t} = \int_t^{\infty} \frac{h(\tau)}{\Pr(t < \tilde{\tau})} \left(\int_t^{\tau} G_2(x)e^{-rx}dx + C_q^u e^{-r\tau} \right) d\tau - R_q e^{-rt}$ for all $q, u \in \{g, s\}$, $q \neq u$. In consequence, a type-*s* firm would force a type-*g* firm out of a growing market if both coincided at some date $t \leq c_g^s$. Intuitively, a firm with a specific asset is more committed to outlast a type-*g* firm because its outside option is less valuable (i.e., $R_s < R_g$) *and* the opportunity cost of exiting the market is (weakly) larger for a type-*s* firm because $C_s^g \geq C_g^s$.

To solve for the equilibrium outcome of the game, it only remains to consider incentives to preempt the competitor when investing, taking into account that $c_s^g < c_g^s$. If the preemptor wanted to acquire a specific asset, then, given that it could not be forced out, it should enter at p_s^g by the REP, thus getting $L_s^g(p_s^g) = F_g^s(f_g^s)$. If the preemptor wanted to invest in a general-purpose factor, it would consider two different situations that are possible. In the first place, if $c_g^s \leq p_g^s$, a first-entrant would have no incentives to enter before p_g^s. As a result, if it entered with a non-specific resource, then it could not be forced out later on, and thus should get $L_g^s(p_g^s) = F_s^g(f_s^g)$. Hence, when $c_g^s \leq p_g^s$, both firms could attain a larger payoff by "tacitly agreeing" to wait until p_g^s (since $F_s^g(f_s^g) \geq F_g^s(f_g^s)$). Of course, the incentive compatibility constraint for such collusive agreement to hold would be that $L_s^g(p_g^s) \leq L_g^s(p_g^s)$. If this held, then no firm would have incentives to enter at p_g^s with a specialized asset, since it would get a smaller payoff, and thus, in equilibrium, one firm would enter with a flexible asset at p_g^s and the other with a specific one at f_s^g. This situation is represented in Figure 3. However, if the constraint did not hold, then the existence of incentives to deviate would rule out such situation as an equilibrium outcome. Hence, in an MPE one firm would invest in a specialized asset at p_s^g, and the rival would enter at f_g^s with a general-purpose resource, provided $L_s^g(p_s^g) \geq L_g^s(p_s^g)$. Otherwise, no symmetric MPE exists, since the first-entrant would prefer to enter with a flexible asset rather than with a specific one.

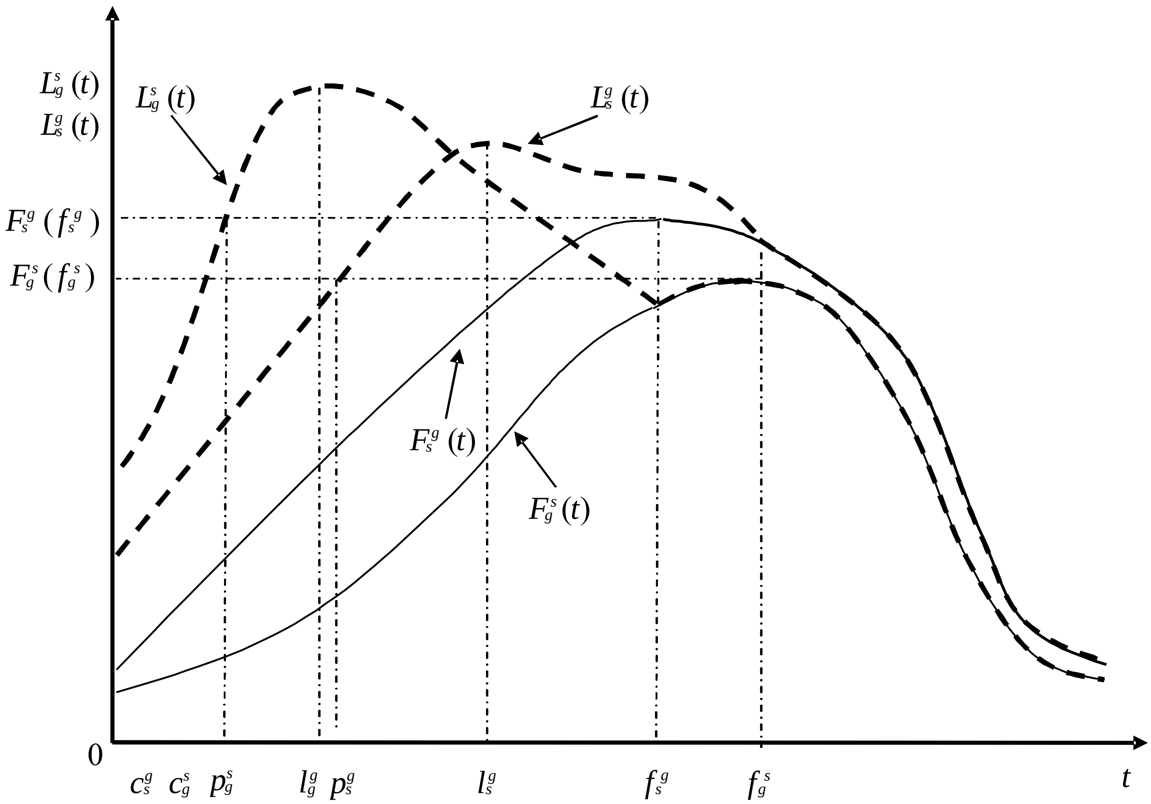

Figure 3

In the second and last place, it remains to study what happens if $p_g^s < c_g^s$, recalling that if the first-entrant invested in a specialized asset, then it should enter at p_s^g so as to avoid being preempted, thus attaining a payoff of $L_s^g(p_s^g) = F_g^s(f_g^s)$. It is clear that the preemptor would enter with a flexible resource at $p_g^s < c_g^s$ if and only if the following two conditions held at the same time: it would not be forced out at $\min(c_g^s, \max(p_g^s, l_s^g))$ (that is, if $L_s^\phi(\min(c_g^s, \max(p_g^s, l_s^g))) \leq F_s^g(f_s^g)$), and it would have no incentives to choose the other type of asset (i.e., $L_s^g(p_g^s) \le L_g^s(p_g^s)$). Given that the satisfaction of the first condition implies that the second one is met,[19] we have the following: if $L_s^\phi(\min(c_g^s, l_s^g)) > L_g^s(p_g^s)$ and/or $L_s^g(p_g^s) > L_g^s(p_g^s)$), then one firm enters at p_s^g with a specific factor, while the other enters later at f_g^s with a general-purpose asset (see Figure 4); if $L_s^\phi(\min(c_g^s, l_s^g)) \le L_g^s(p_g^s)$, then one firm invests in a flexible resource at p_g^s, and the other invests in a specialized one at f_g^s.

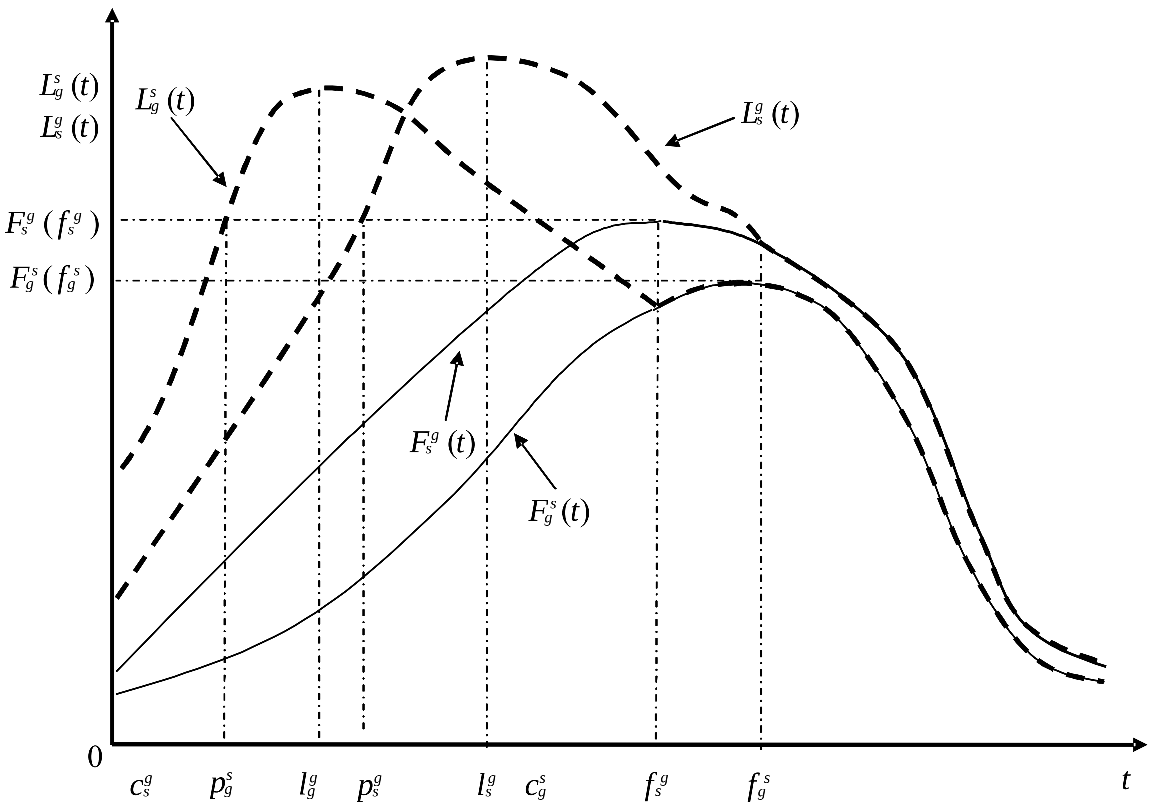

Figure 4

We can now summarize and state the main result when $C_s^g \ge C_g^g$:

[19] First note that $p_g^s < l_g^s < l_s^g$, which, together with the fact that $p_g^s < c_g^s$, implies that $p_g^s < \min(c_g^s, l_s^g)$. On the one hand, if $c_g^s < l_s^g$, the first condition boils down to $L_s^g(c_g^s) < L_s^\phi(c_g^s) \le F_s^g(f_s^g) = L_g^s(p_g^s)$. Because $c_g^s \in (p_g^s, l_s^g)$, $L_s^g(p_g^s) < L_s^g(c_g^s)$, so the first condition is indeed stronger than the second one. On the other hand, if $l_s^g \le c_g^s$, then $L_s^g(l_s^g) < L_s^\phi(l_s^g) \le F_s^g(f_s^g) = L_g^s(l_s^g)$, and it is clear that satisfaction of the first condition implies automatic satisfaction of the second, since $L_g^s(t) - L_s^g(t)$ is a decreasing function because $C_g^\phi > C_s^\phi$.

Proposition 5: Let $C_g^s \leq C_s^g$, and distinguish two cases:

(i) If $L_s^{\phi}(\min(l_s^g, \max(c_g^s, p_g^s))) > L_g^s(p_g^s)$ and/or $L_s^g(p_g^s) > L_g^s(p_g^s))$, then, with probability one-half, one of the firms enters with a specialized plant at p_s^g and the other enters with a multipurpose plant at f_g^s, while with probability one-half the identities of the firms are interchanged. Each attains the same equilibrium payoff: $L_s^g(p_s^g) = F_g^s(f_g^s)$.

(ii) If $L_s^{\phi}(\min(l_s^g, \max(c_g^s, p_g^s))) \leq L_g^s(p_g^s)$, then, with probability one-half, one of the firms enters with a multipurpose plant at p_g^s and the other enters with a specialized plant at f_s^g, while with probability one-half the identities of the firms are interchanged. Each attains the same equilibrium payoff: $L_g^s(p_g^s) = F_s^g(f_s^g)$.

Proposition 4 already deals with comparative statics on an *s*-*g* equilibrium. The following result deals with those equilibrium situations in which the first-entrant invests in a multipurpose plant and the second-entrant invests in a specialized plant:

Proposition 6: In a *g*-*s* equilibrium, as R_g increases, both firms' equilibrium payoff increases, while the second-entrant speeds up its entry time. An increase in R_s increases equilibrium payoffs too, and speeds up both firms' investment.

Proof: See Appendix.■

Both an increase in R_g and R_s augment the continuation payoff of a type-*s* firm that coincides with a type-*g* firm in a declining market. As a result, the second-entrant speeds up its entry time, and gets a higher payoff, thereby increasing the first-entrant's equilibrium payoff as well. Finally, a higher R_s diminishes the payoff of a firm that enters in the first place with a general-purpose asset (because of the decrease in f_s^g), and increases at the same time the value of entering in the second place with a specialized factor. Consequently, the first-entrant delays the date at which it preempts its rival firm and thus gets a larger payoff to equalize that attained by the competitor.

5. CONCLUSION

In this paper, we have studied a model in which two firms choose not only when to lead/follow, but also how to lead/follow. Specifically, firms can choose to invest in a specialized or a general-purpose asset when entering an industry. We have shown that both firms are generally expected to enter with flexible resources when a firm that would operate a non-specific asset would gain a higher payoff than a firm with a specialized asset throughout the decline of a duopolistic industry. However, if this condition does not hold, firms usually choose to invest in distinct assets: the first entrant enters with a specialized factor and the second entrant invests in a non-specific resource. We have also shown that an increase in the redeployment values of any type of asset increases the equilibrium payoff of each firm, and speeds up the second-mover's entry, irrespective of the type of asset in which it chooses to invest.

This paper has assumed that the (undiscounted) value of the outside option is uncorrelated to market evolution. This could be for example the case of a multinational firm that bears a country-specific shock and transfers resources to another country in which they are more valuable. There are situations, though, in which at least the opportunity cost of operating a general-purpose asset is related to the evolution of the market in consideration. Although it is probably desirable to take this feature into account, we hope it is clear that the main ingredients and implications of our analysis would hold in such setting.

Lastly, we would like to stress broader implications of our framework from a conceptual and predictive standpoint, since it is formally equivalent to setups that may seem very different. For instance, we could have assumed that firms have to choose the type of organizational structure rather than the type of asset with which to operate. Thus, a "rigid" organizational structure would imply a lag when trying to implement strategic decisions such as exit, despite the opportunity cost of being active could be the same as that of a "flexible" organizational structure which would allow for a rapid response to changes. In this sense, this paper has shown that, under certain circumstances, it may be advantageous to be a "dinosaur."

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APPENDIX

Proof of Proposition 1: Suppose that a type-q firm $(q \in \{g, s\})$ chooses to exit while alone in the market, conditional upon the market still growing at that time. In order to decide its exit time t' given information at date t, the firm solves the following program:

$$\max_{t' \geq t} X_q^{\phi}(t') = \int_t^{t'} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_t^{\tau} G_1(x) e^{-rx} dx + C_q^{\phi} e^{-r\tau} \right) d\tau +$$
$$\int_{t'}^{\infty} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_t^{t'} G_1(x) e^{-rx} dx + R_q e^{-rt'} \right) d\tau.$$

If the realization of $\tilde{\tau}$ falls into the set $[t, t']$, then a type-q firm gains a monopoly profit over the entire lifetime of the industry. If the realization is greater than t', the firm can safely exit at t'. Differentiating $X_q^{\phi}(t')$ with respect to t' noting that $h(t') = \lambda e^{-\lambda t'}$ and $\int_{t'}^{\infty} h(\tau) d\tau = e^{-\lambda t'}$ yields that $\frac{dX_q^{\phi}(t')}{dt'} = e^{-(r+\lambda)t'}[G_1(t') + \lambda C_q^{\phi} - (r+\lambda)R_q]$. If $C_q^{\phi} < \frac{(r+\lambda)R_q - G_1(0)}{\lambda}$, setting $\frac{dX_q^{\phi}(t')}{dt'} = 0$ and rearranging yields the unique candidate for a minimum: $x_q^{\phi} = G_1^{-1}((r+\lambda)R_q - \lambda C_q^{\phi}) > 0$. The inequality follows from the strict increasingness of $G_1(\cdot)$. Let us check that the minimum is attained at x_q^{ϕ}, that is, $\frac{d^2 X_q^{\phi}(x_q^{\phi})}{dt'^2} > 0$. $\frac{d^2 X_i^1(t_i^1/t)}{dt'^2} > 0$. Since $\frac{dX_q^{\phi}(t')}{dt'} = e^{-(r+\lambda)t'}[G_1(t') + \lambda C_q^{\phi} - (r+\lambda)R_q]$, denoting derivatives by primes, it follows that $\frac{d^2 X_q^{\phi}(x_q^{\phi})}{dt'^2} = -e^{-(r+\lambda)x_q^{\phi}}\{(r+\lambda)[G_1(x_q^{\phi}) + \lambda C_q^{\phi} - (r+\lambda)R_q] - G_1'(x_q^{\phi})\} > 0$, given that $G_1(x_q^{\phi}) = (r+\lambda)R_q - \lambda C_q^{\phi}$ and $G_1'(\cdot) > 0$. If $C_q^{\phi} \geq \frac{(r+\lambda)R_q - G_1(0)}{\lambda}$, then let $x_q^{\phi} = 0$. This shows that $X_q^{\phi}(t')$ is strictly increasing on $t \geq x_q^{\phi}$, and thus it follows that a type-q firm is not willing to exit a growing market whenever it is a monopolist at $t \geq x_q^{\phi}$.

Now let T_q^ϕ denote the set of dates at which a type-q firm $(q \in \{g,s\})$ is not willing to exit when the rival is inactive. By the previous results, T_q^ϕ is not empty for $q \in \{g,s\}$. The next step of the proof requires showing that $\hat{t} \in T_q^\phi$ for all $\hat{t} > t$ whenever $t \in T_q^\phi$. Suppose to the contrary that $t \in T_q^\phi$, but $\hat{t} \notin T_q^\phi$ for some $\hat{t} > t$. In particular, let $\hat{t} = \inf\{s > t : s \notin T_q^\phi\}$, with $\hat{t} \notin T_q^\phi$.[20] By definition, a type-q firm would exit immediately at $\hat{t}$ (it could be indifferent between immediate exit and no exit, so, in case of indifference, let us assume that it chooses to exit rather than remain active). Now consider date $\hat{t} - \varepsilon$, for $\varepsilon > 0$ small enough. At such date, a type-q firm prefers to remain active if the market is growing, again by definition of $\hat{t}$. But, given that it plans to exit almost immediately at $\hat{t}$, it can be proven that the firm would be better off exiting, which would entail a contradiction since $\hat{t} - \varepsilon$ belongs to its no exit region. Thus, if a type-q firm that is active at $\hat{t} - \varepsilon$ waits to exit until $\hat{t}$, then it expects to gain:

$$X_q^\phi(\hat{t}) = \frac{1}{\Pr(\hat{t} - \varepsilon < \tilde{\tau})}\left[\int_{\hat{t}-\varepsilon}^{\hat{t}} h(\tau)\left(\int_{\hat{t}-\varepsilon}^{\tau} G_1(x)e^{-rx}dx + C_q^\phi e^{-r\tau}\right)d\tau + \left(\int_{\hat{t}}^{\infty} h(\tau)d\tau\right)\left(\int_{\hat{t}-\varepsilon}^{\hat{t}} G_1(x)e^{-rx}dx + R_q e^{-r\hat{t}}\right)\right]$$

$$= \frac{(\varepsilon G_1(\hat{t})e^{-r\hat{t}} + R_q e^{-r\hat{t}})\int_{\hat{t}}^{\infty} h(\tau)d\tau + \varepsilon^2 h(\hat{t})G_1(\hat{t})e^{-r\hat{t}} + \varepsilon C_q^\phi h(\hat{t})e^{-r\hat{t}}}{(1+\lambda\varepsilon)e^{-\lambda\hat{t}}} =$$

$$= \varepsilon G_1(\hat{t})e^{-r\hat{t}} + \frac{e^{\lambda\hat{t}}(R_q e^{-(r+\lambda)\hat{t}} + \varepsilon\lambda C_q^\phi e^{-(r+\lambda)\hat{t}})}{1+\lambda\varepsilon},$$

using the facts that $e^{\rho\varepsilon} = 1 + \rho\varepsilon$ for small $\varepsilon > 0$ and $h(\tau) = \lambda e^{-\lambda\tau}$. By immediately exiting at $\hat{t} - \varepsilon$, a type-q firm would seize $R_q e^{-r(\hat{t}-\varepsilon)} = R_q e^{-r\hat{t}} + \varepsilon r R_q e^{-r\hat{t}}$. We claim that, for sufficiently small $\varepsilon > 0$, this payoff is greater than $X_q^\phi(\hat{t})$ if a type-q firm is active at $\hat{t} - \varepsilon$. Let us suppose to the contrary that

[20] If $\hat{t} \in T_q^\phi$, just reverse the roles of $\hat{t}$ and $\hat{t} - \varepsilon$ in the proof, and let $\varepsilon < 0$ be sufficiently large.

$R_q e^{-r\hat{t}} + \varepsilon r R_q e^{-r\hat{t}} \le \varepsilon G_1(\hat{t}) e^{-r\hat{t}} + \dfrac{e^{\lambda\hat{t}}(R_q e^{-(r+\lambda)\hat{t}} + \varepsilon\lambda C_q^{\phi} e^{-(r+\lambda)\hat{t}})}{1+\lambda\varepsilon}$. Multiplying through by $\dfrac{e^{r\hat{t}}(1+\lambda\varepsilon)}{\varepsilon} > 0$, canceling terms, and letting $\varepsilon \to 0$ yields $(r+\lambda)R_q \le G_1(\hat{t}) + \lambda C_q^{\phi}$, so $\hat{t} \ge x_q^{\phi}$, which contradicts the fact that a type-*q* firm is not willing to exit a growing market if it is a monopolist at any $\hat{t} \ge x_q^{\phi}$.

Now let $[0,\infty) \setminus T_q^{\phi}$ be the immediate exit region for a type-*q* firm when it is alone in the market. If the set $[0,\infty) \setminus T_q^{\phi}$ is empty, then the proof of the proposition is finished. If it is not, then the fact that $K > R_q$ for all $q \in \{g,s\}$ implies that a type-*q* firm has no incentives to enter the market at any $t \in [0,\infty) \setminus T_q^{\phi}$, given that it will exit immediately after entering. As a result, any firm that finds it optimal to enter a growing market as a monopolist does not exit whilst the (expanding) industry is monopolistic.■

Proof of Proposition 2: If a type-*q* firm coincides in a growing market with a type-*u* firm $(q,u \in \{g,s\}, q \ne u)$, then it solves:

$$\max_{t' \ge t} X_q^u(t') = \int_t^{t'} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_t^{\tau} G_2(x) e^{-rx} dx + C_q^u e^{-r\tau} \right) d\tau + \int_{t'}^{\infty} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_t^{t'} G_2(x) e^{-rx} dx + R_q e^{-rt'} \right) d\tau.$$

Like $X_q^{\phi}(t')$, $X_q^u(t')$ is a strictly quasi-convex function with a minimum at $x_q^u = G_2^{-1}[(r+\lambda)R_q - \lambda C_q^u]$ (since $G_2(\cdot)$ is strictly increasing), so the optimizer of the program is $\max(t, x_q^u)$. This shows that $X_q^u(t')$ is strictly increasing on $t \ge x_q^u$, and thus it follows that a type-*q* firm competing against a type-*u* rival is not willing to exit a growing market at any time $t \ge x_q^u$. Following identical steps to those in the proof of Proposition 1, and letting T_q^u denote the region on which a type-*q* firm has no (stand-alone) incentives to exit, it is easy to show that $\hat{t} \in T_q^u$ for all $\hat{t} > t$ whenever $t \in T_q^u$. Let c_q^u be the latest date at which a type-*q* firm prefers exiting rather than staying in a growing market. Formally:

$$c_q^u = \inf T_q^u = \inf\left\{t \in [0,\infty) : R_q e^{-rt} \le \int_t^{\infty} \frac{h(\tau)}{\Pr(t<\tilde{\tau})}\left(\int_t^{\tau} G_2(x)e^{-rx}dx + C_q^u e^{-r\tau}\right)d\tau\right\}.$$

The proposition clearly follows for $t > \max(c_s^g, c_g^s),$ as we have that for all $q,u \in \{g,s\}$:

$$R_q e^{-rt} = X_q^u(t/t) < \max_{t' \ge t} X_q^u(t'/t) = X_q^u(\infty/t) = \int_t^{\infty} \frac{h(\tau)}{\Pr(t<\tilde{\tau})}\left(\int_t^{\tau} G_2(x)e^{-rx}dx + C_q^u e^{-r\tau}\right)d\tau.$$

Also, if $\min(c_s^g, c_g^s) < t \le \max(c_s^g, c_g^s),$ then it is dominant for one of the firms to remain in the market while it is expanding and, consequently, its rival exits immediately at *t* since it is on its exit region.

Therefore, to conclude the proof, it only remains to examine what happens if $t \le \min(c_s^g, c_g^s).$ For such a date, a firm's incentives to stay in the market depend on whether it can force its rival out in the war of attrition that would take place if both remained active. For $q \in \{g,s\},$ let $\bar{t}_q^{\phi}$ be such that $R_q e^{-r\bar{t}_q^{\phi}} = \int_{\bar{t}_q^{\phi}}^{\infty} \frac{h(\tau)}{\Pr(\bar{t}_q^{\phi} < \tilde{\tau})}\left(\int_{\bar{t}_q^{\phi}}^{\tau} G_1(x)e^{-rx}dx + C_q^{\phi} e^{-r\tau}\right)d\tau,$ with $\bar{t}_q^{\phi} = 0$ if it does not exist. (If it exists, then uniqueness follows from the same arguments that show the uniqueness of c_q^u. Also, it is straightforward to show that $\bar{t}_q^{\phi} < c_q^u$.) Let us now focus on dates $t \in [\max_{q\in\{g,s\}} \bar{t}_q^{\phi}, \min(c_s^g, c_g^s)],$ since at any previous late, at least one firm would exit even if it were alone in the market at that time. We now show that these situations fall into the category of (nonstationary) wars of attrition with eventual continuation analyzed by Fudenberg and Tirole (1991, pp. 121-125), whence the desired result follows. So suppose that a type-*g* firm coincides with a type-*s* firm in a growing market at date $t > 0.$ If the rival of a type-*q* firm –whose type is hence $u \ne q \in \{g,s\}$– exits first at $t' \ge t,$ then the payoff of the type-*q* firm as a function of its competitor's exit date is:

$$\bar{F}_q^u(t') = \int_t^{t'} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_t^{\tau} G_2(x) e^{-rx} dx + C_q^u e^{-r\tau} \right) d\tau +$$

$$\int_{t'}^{\infty} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_t^{t'} G_2(x) e^{-rx} dx + \int_{t'}^{\tau} G_1(x) e^{-rx} dx + C_q^{\phi} e^{-r\tau} \right) d\tau.$$

It is easy to see that this function is downward sloping, since:

$$\frac{d\bar{F}_q^u(t')}{dt'} = \Pr(t' < \tilde{\tau}) e^{-rt'} [\lambda (C_q^u - C_q^{\phi}) + G_2(t') - G_1(t')] < 0.$$

If the type-q firm exits first at $t' \geq t,$ then its payoff is as follows:

$$\bar{L}_q^u(t') = \int_t^{t'} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_t^{\tau} G_2(x) e^{-rx} dx + C_q^u e^{-r\tau} \right) d\tau +$$

$$\int_{t'}^{\infty} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_t^{t'} G_2(x) e^{-rx} dx + R_q e^{-rt'} \right) d\tau.$$

The derivative of this function is:

$$\frac{d\bar{L}_q^u(t')}{dt'} = \Pr(t' < \tilde{\tau}) e^{-rt'} [G_2(t') + \lambda C_q^u - (r + \lambda) R_q)].$$

This function is strictly quasi-convex, attaining a minimum at x_q^u. Furthermore, $\bar{F}_q^u(\infty) = \bar{L}_q^u(\infty),$ $\bar{L}_q^u(t) \geq \bar{L}_q^u(\infty)$ given that $t \leq \min(c_s^g, c_g^s),$ and $\bar{F}_q^u(t') - \bar{L}_q^u(t') = \int_{t'}^{\infty} \left(\frac{h(\tau)}{\Pr(t < \tilde{\tau})} \right) \left(\int_{t'}^{\tau} G_1(x) e^{-rx} dx + C_q^{\phi} e^{-r\tau} - R_q e^{-rt'} \right) d\tau > 0$ because $t \geq \max_{q \in \{g,s\}} \bar{t}_q^{\phi}.$

Note that the properties of these functions hold for any arbitrary $q, u \in \{g, s\}$ (with $q \neq u$), so it follows that the type-s firm loses the war of attrition and exits at date t if $c_g^s < c_s^g,$ while the type-g firm is the loser and exits at t if $c_s^g < c_g^s.$ ■

Proof of Proposition 4: The envelope theorem implies that $\frac{dF_g^g(f_g^g(R_g), R_g)}{dR_g} > 0$ because $\frac{dC_g^g}{dR_g} > 0,$ which also implies that $\frac{df_g^g}{dR_g} < 0,$ whence the claim about the g-g

equilibrium follows. As for the *s*-*g* equilibrium, note that $\frac{df_g^s}{dR_s} = 0 = \frac{dF_g^s(f_g^s(R_s), R_s)}{dR_s}$. In addition, total differentiation of $L_s^g(p_s^g(R_s), R_s) = F_g^s(f_g^s(R_s), R_s)$ using the facts that $\frac{dF_g^s(f_g^s(R_s), R_s)}{dR_s} = 0$, $\frac{dC_s^\phi}{dR_s} = \frac{dC_s^g}{dR_s}$ and $G_1(l_s^g) = (r+\lambda)K - \lambda C_s^\phi$ yields the following analytical expression for $\frac{dp_s^g}{dR_s}$ after some manipulations:

$$\frac{dp_s^g}{dR_s} = \left(\frac{\lambda}{(r+\lambda)[G_1(p_s^g) - G_1(l_s^g)]}\right)\left(\frac{dC_s^\phi}{dR_s}\right).$$

So $\frac{dp_s^g}{dR_s} < 0$ because $\frac{dC_s^\phi}{dR_s} > 0$ and $p_s^g < l_s^g$.

Similarly, totally differentiating $L_s^g(p_s^g(R_g), R_g) = F_g^s(f_g^s(R_g), R_g)$ taking into account that $C_s^\phi = C_s^g$ and $G_1(l_s^g) = (r+\lambda)K - \lambda C_s^\phi$ implies that:

$$\frac{dp_s^g}{dR_g} = \frac{\int_{f_g^s}^{\infty} h(\tau)e^{-r\tau}\left(\frac{dC_g^s}{dR_g}\right)d\tau - \frac{df_g^s}{dR_g}\left\{[G_1(f_g^s) - G_2(f_g^s)]e^{-(r+\lambda)p_g^s}\right\}}{[G_1(p_s^g) - G_1(l_s^g)]e^{-(r+\lambda)p_s^g}}.$$

So $\frac{dp_s^g}{dR_g} > 0$ because $\frac{dC_g^s}{dR_s} > 0$, $l_s^g > p_s^g$, and $\frac{df_g^s}{dR_g} < 0$, which follows because $\frac{dC_g^s}{dR_g} > 0$. This also implies that $\frac{dF_g^s(f_g^s(R_g), R_g)}{dR_g} > 0$. ■

Proof of Proposition 6: First, $\frac{df_s^g}{dR_g} < 0$ because $\frac{dC_s^g}{dR_g} > 0$, which, together with an application of the envelope theorem implies that $\frac{dF_s^g(f_s^g(R_g), R_g)}{dR_g} > 0$. Also, note that $\frac{dC_s^g}{dR_s} > 0$ implies that $\frac{df_s^g}{dR_s} < 0$ and $\frac{dF_s^g(f_s^g(R_s), R_s)}{dR_s} > 0$ (again by the envelope theorem). Finally, total differentiation of $L_g^s(p_g^s(R_s), R_s) = F_s^g(f_s^g(R_s), R_s)$ with respect to R_s using the envelope theorem, and straightforward manipulations taking into account that $\frac{dC_g^{\phi}}{dR_s} = 0 = \frac{dC_g^s}{dR_s}$ yield the following:

$$\frac{dp_g^s}{dR_s} = \frac{\int_{f_s^m}^{\infty} h(\tau) e^{-r\tau} \left(\frac{dC_s^g}{dR_s} \right) d\tau - \frac{df_s^g}{dR_s} \{ [G_1(f_s^g) - G_2(f_s^g) + \lambda(C_g^{\phi} - C_g^s)] e^{-(r+\lambda) f_s^g} \}}{[G_1(l_g^s) - G_1(p_g^s)] e^{-(r+\lambda) p_g^s}}.$$

Given that $\frac{dC_s^g}{dR_s} > 0$, $C_g^{\phi} > C_g^s$, $\frac{df_s^g}{dR_s} < 0$ and $p_g^s < l_g^s$, it follows that $\frac{dp_g^s}{dR_s} > 0$. ■